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Stability analysis of discrete-time positive polynomial-fuzzy-model-based control systems through fuzzy co-positive Lyapunov function with bounded control

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Abstract: This study employs a novel fuzzy co-positive Lyapunov function to investigate the stability of discrete-time polynomial-fuzzy-model-based control systems under positivity constraint. The fuzzy co-positive Lyapunov function consists of a number of local sub-Lyapunov function candidates which includes the positivity property of a non-linear system and the contribution of each sub-Lyapunov function candidates depends on the corresponding membership functions. Imperfect premise matching design concept is used for the design of a closed-loop polynomial fuzzy controller based on the constructed polynomial fuzzy model. The bounded control signal conditions (upper and lower boundary demands on control signal) are included in the Lyapunov stability and positivity conditions, in which all are formulated in the form of sum-of-squares conditions. A numerical example is given to validate the proposed approach.

1 Introduction

One can find examples in chemical reactors, storage systems and drug-delivery, wherein the mathematical models of the system states' response to initial positive condition is always confined in the non-negative orthant space. We call these systems positive systems. There is a great deal of research in the literature on the fundamental properties of these systems, i.e. positivity, controllability, stability and observability [1–9]. In the modelling of positive non-linear systems, the traditional Takagi–Sugeno (T-S) fuzzy model [1–9] offers an effective and systematic way to represent the dynamics of positive non-linear systems using local linear systems weighed by corresponding membership functions [10–15]. Traditional linear T-S Fuzzy control and stability analysis techniques have been further exploited in T-S fuzzy-model-based (FMB) positive system [16–22]. Typically, Lyapunov stability and positivity conditions formulated in linear matrix inequalities (LMIs) are obtained to synthesise fuzzy controller and guarantee the stability and positivity of the closed-loop T-S FMB control systems [16–18]. Besides, filters are designed for positive T-S FMB control systems in [20, 23] and positivity and stability analysis of special types of positive systems including positive switch systems and positive Markovian jump systems are also investigated in the literature [24–30].

Despite recent developments, further, improvement needs to be made for the analysis of fuzzy positive systems. In modelling, polynomial fuzzy models have been suggested so far to represent the dynamics of general non-linear positive systems more accurately using local polynomial systems weighed by the corresponding membership functions [31–35]. The capability of premise modelling of non-linear systems is largely improved compared with the traditional T-S fuzzy models. Owing to the fact that the existence of non-linear polynomial elements in the model would not be represented with certain constants as it is normally done in the original T-S fuzzy models. As a result, the number of fuzzy rules can be largely decreased and the structure of overall fuzzy model is simplified. In this paper, we increase the accuracy of system approximation by using polynomial fuzzy model to represent positive non-linear systems. For the analysis, LMI solver has proved incapable of solving stability and positivity feasibility problems with polynomial terms. Therefore, we formulate the stability and positivity condition by sum-of-square (SOS) forms [31–33, 36, 37], which can be easily solved through a MATLAB® third-party toolbox SOSTOOLS.

Using the controller design approach of perfect premise matching, the shape and number of fuzzy rules in the model and controller must be exactly matched [16–22]. Even though this design approach eases the stability analysis, it limits the flexibility of fuzzy controller design and increases implementation cost. To overcome this drawback, imperfect premise matching design approach has been suggested [32, 38, 39] where the shape and number of controller rules are allowed to be chosen freely and independently of the model. In this way, the shape of membership functions is simplified and the number of rules can be substantially reduced in the fuzzy controller [40]. We also choose this design approach for the polynomial fuzzy controller in this paper.

To enhance the feasibility of the stability analysis of positive discrete-time polynomial-FMB (PFMB) control system, we further relax the stability conditions using membership-function-dependent stability analysis [40], and different types of Lyapunov function candidates. In the previous studies, the authors suggested approaches to reduce the conservativeness of positive non-linear system stability conditions by including the information of membership functions in the feasibility conditions [41–43]. In [41, 43–45], approximated membership functions were proposed to represent the approximation error between the approximated and original membership functions in stability conditions and in [42] the membership functions are treated as symbol variables in stability conditions to relax the conservativeness. Quadratic Lyapunov function is often employed for the stability analysis of positive system due to its generality [19, 22]. A more favourable form of Lyapunov function called linear co-positive Lyapunov function (LCLF) was also attempted for the stability analysis of positive discrete-time PFMB control system [16–18]. LCLF naturally hold the positivity property of a non-linear system and when using with non-negative vectors for the stability analysis of positive discrete-time PFMB control system, it shall be defined as constants to hold the positivity property of Lyapunov functions

[41–43]. This eases the stability analysis using different time-variable terms; however, it limits the generality of LCLF formulation. In this paper, a fuzzy co-positive Lyapunov function is employed to study stability analysis of positive discrete-time PFMB control system to relax positivity, stability and bounded control signal conditions. Bounded control, by definition, means the control signal must be limited by a lower and upper boundary. There can be found a number of practical systems [6, 46–48] with bounded control. In the design of the fuzzy controllers, the bounds of control signal are also taken into account while in most cases the original operating domain would be unwantedly limited as well [6, 46–48]. In this paper, the boundary of operating domain is considered along with designing bounded control for discrete-time PFMB control system not to restrict the original operating domain. The contributions of this paper are briefly as follows:

(i) The stability and positivity analysis is formulated for discrete-time PFMB control system under imperfect premise matching design concept.
(ii) The fuzzy co-positive Lyapunov function is proposed for the stability of discrete-time positive PFMB control system with the non-negative vectors in the constant form. This extends the formulation to more general non-negative vectors in the polynomial form which improves the generality of Lyapunov function form and reduces the conservativeness of positivity, stability and bounded control conditions.
(iii) Polynomial fuzzy controller is synthesised to ensure the stability and positivity of non-linear system and the certain bounds of the control signal are considered to guarantee the values of control signal confined to a certain range without effecting original operating domain region.

The structure of this paper is organised as follows. In Section 2, the commonly used notations in this paper are introduced. In Section 3, the formulation of polynomial fuzzy model and controller under imperfect premise matching design concept are presented. In Section 4, the stability and positivity analysis based on fuzzy co-positive Lyapunov function are formulated considering bounded controls. In Section 5, a simulation example is given to verify the proposed method. Finally, Section 6 concludes the paper.

2 Notations

The following notations are adopted throughout this paper. A monomial in $\mathbf{x}(k) = [x_1(k), x_2(k), \dots, x_n(k)]$ is a function of the form $x_1^{d_1}(k), x_2^{d_1}(k), \dots, x_n^{d_n}(k)$, where d_i, $i \in \{1, 2, \dots, n\}$ is a non-negative integer. The degree of a monomial is defined as $d = \sum_{i=1}^{n} d_i$. $\mathbf{p}(\mathbf{x}(k))$ is a polynomial if it can be expressed as a finite linear combination of monomials with real coefficients. $\mathbf{p}(\mathbf{x}(k)) = \sum_{j=1}^{m} \mathbf{q}_j(\mathbf{x}(k))^2$ indicates the polynomial $\mathbf{p}(\mathbf{x}(k))$ is an SOS implying $\mathbf{p}(\mathbf{x}(k)) \geq 0$ where $\mathbf{q}_j(\mathbf{x}(k))$ is a polynomial and m is a non-zero positive integer. $\mathbf{Z} \succ 0$, $\mathbf{Z} \succcurlyeq 0$, $\mathbf{Z} \prec 0$ and $\mathbf{Z} \preccurlyeq 0$ mean that all the elements of the matrix $\mathbf{Z}$ are positive, semipositive, negative and seminegative, respectively. $\mathbf{R}^{\mathrm{T}}$ stands for the transpose of a real matrix $\mathbf{R}$. $\underline{n} = \{1, 2, \dots, n\}$ where $n \in \mathbb{Z}^+$ denotes the order of fuzzy model. $\underline{p} = \{1, 2, \dots, p\}$ where $p \in \mathbb{Z}^+$ is the rule number of fuzzy model. We define the normalised grade of membership of fuzzy model and fuzzy controller as $\mathbf{w}(\mathbf{x}(k)) = [w_1(\mathbf{x}(k)), w_2(\mathbf{x}(k)), \dots, w_i(\mathbf{x}(k))], i \in \underline{p}$ and $\mathbf{m}(\mathbf{x}(k)) = [m_1(\mathbf{x}(k)), m_2(\mathbf{x}(k)), \dots, m_j(\mathbf{x}(k))], j \in \underline{c}$, respectively. In addition vector variables in this paper are denoted by bold font.

3 Preliminaries

3.1 Discrete-time polynomial fuzzy models

We assume that the dynamics of non-linear discrete-time systems can be represented by p fuzzy rules of discrete-time polynomial fuzzy model where the ith rule is presented as follows:

$$\begin{aligned} &\text{Rule } i: \quad \text{IF } f_1(\mathbf{x}(k)) \text{ is } M_1^i \text{ AND} \cdots \text{AND } f_\Psi(\mathbf{x}(k)) \text{ is } M_\Psi^i \\ &\text{THEN } \mathbf{x}(k+1) = \mathbf{A}_i(\mathbf{x}(k))\mathbf{x}(k) + \mathbf{B}_i(\mathbf{x}(k))\mathbf{u}(k) \end{aligned} \tag{1}$$

$$\mathbf{x}(0) = \boldsymbol{\phi}(0), \tag{2}$$

where M_l^i, $l = \{1, 2, \dots, \Psi\}$ is the fuzzy set of i corresponding to premise variable $f_l(\mathbf{x}(k))$, with $l = \{1, 2, \dots, \Psi\}$; Ψ is a positive integer; $\boldsymbol{\phi}(0)$ is the vector valued initial function; $\mathbf{A}_i(\mathbf{x}(k)) \in \Re^{n \times n}$ and $\mathbf{B}_i(\mathbf{x}(k)) \in \Re^{n \times m}$ are polynomial system and input matrices, respectively, with $i \in \underline{p} = \{1, 2, \dots, p\}$, p is the number of IF-THEN rules; $\mathbf{x}(k) \in \Re^n$ and $\mathbf{u}(k) \in \Re^m$ are state vector and control input vector, respectively.

The dynamics of non-linear systems is described as follows:

$$\mathbf{x}(k+1) = \sum_{i=1}^{p} w_i(\mathbf{x}(k))(\mathbf{A}_i(\mathbf{x}(k))\mathbf{x}(k) + \mathbf{B}_i(\mathbf{x}(k))\mathbf{u}(k)), \tag{3}$$

where

$$\sum_{i=1}^{p} w_i(\mathbf{x}(k)) = 1, \tag{4}$$

$$w_i(\mathbf{x}(k)) = \frac{\prod_{l=1}^{\Psi} \mu_{M_l^i}(f_l(\mathbf{x}(k)))}{\sum_{n=1}^{p} \prod_{l=1}^{\Psi} \mu_{M_l^n}(f_l(\mathbf{x}(k)))} \quad i \in \underline{p}, \tag{5}$$

$$w_i(\mathbf{x}(k)) \geq 0 \quad i \in \underline{p}, \tag{6}$$

where $w_i(\mathbf{x}(k))$ is the normalised grade of membership, $\mu_{M_l^i}(f_l(\mathbf{x}(k)))$ is the grade of membership corresponding to the fuzzy term M_l^i.

Definition 1: System (3) is said to be positive if the initial condition $\boldsymbol{\phi}(0) \succcurlyeq 0$ holds and the corresponding trajectory $\mathbf{x}(k) \succcurlyeq 0$ for all k [16, 19].

Lemma 1: System (3) is said to be positive if the system matrices satisfy the condition that $\mathbf{A}_i(\mathbf{x}(k)) \succcurlyeq 0$ when $\mathbf{u}(k) = \mathbf{0}$ [16, 19].

3.2 Discrete-time polynomial fuzzy controller

Based on imperfect premise matching design concept, the jth rule of the polynomial fuzzy controller is described as follows:

$$\begin{aligned} &\text{Rule } j: \quad \text{IF } g_1(\mathbf{x}(k)) \text{ is } N_1^j \text{ AND} \cdots \text{AND } g_\Omega(\mathbf{x}(k)) \text{ is } N_\Omega^j \\ &\text{THEN } \mathbf{u}(k) = \mathbf{G}_j(\mathbf{x}(k))\mathbf{x}(k), \end{aligned} \tag{7}$$

where N_l^j is the fuzzy set of j corresponding to the premise variable $g_l(\mathbf{x}(k))$, with $l \in \{1, 2, \dots, \Omega\}$, and Ω is a positive integer; $\mathbf{G}_j(\mathbf{x}(k) \in \Re^{m \times n}$ is the polynomial feedback gain with $j \in \underline{c} = \{1, 2, \dots, c\}$, c is the number of IF-THEN rules. Therefore, the discrete-time polynomial fuzzy controller is given as follows:

$$\mathbf{u}(k) = \sum_{j=1}^{c} m_j(\mathbf{x}(k))\mathbf{G}_j(\mathbf{x}(k))\mathbf{x}(k), \tag{8}$$

where

$$\sum_{j=1}^{c} m_j(\mathbf{x}(k)) = 1, \tag{9}$$

$$m_j(\mathbf{x}(k)) = \frac{\prod_{l=1}^{\Omega} \mu_{N_l^j}(g_l(\mathbf{x}(k)))}{\sum_{n=1}^{c} \prod_{l=1}^{\Omega} \mu_{N_l^n}(g_l(\mathbf{x}(k)))}, \quad j \in \underline{c}, \tag{10}$$

$$m_j(\mathbf{x}(k)) \geq 0, \quad j \in \underline{c}, \tag{11}$$

$m_j(\mathbf{x}(k))$ is the normalised grade of membership, $\mu_{N_l^j}(g_l(\mathbf{x}(k)))$ is the grade of membership corresponding to the fuzzy term N_l^j.

Remark 1: Under the imperfect premise matching design concept, the shape of polynomial fuzzy controller's membership function can be chosen freely from those of the polynomial fuzzy model [36, 37].

4 Stability and positivity analysis

In this section, the stability and positivity conditions for discrete-time polynomial fuzzy systems are formulated. First, the formulation of closed-loop discrete-time PFMB control systems is presented. The SOS-based positivity conditions based on Lemma 2 are then formulated. The fuzzy co-positive Lyapunov function is employed to obtain the SOS-based stability conditions and the gains of the discrete-time fuzzy controller. The formulation of a closed-loop discrete-time PFMB control system is described based on the model (3) and the controller (8) as follows:

$$\begin{aligned}\mathbf{x}(k+1) = &\sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{x}(k))m_j(\mathbf{x}(k))\big((\mathbf{A}_i(\mathbf{x}(k))\\ &+\mathbf{B}_i(\mathbf{x}(k))\mathbf{G}_j(\mathbf{x}(k)))\mathbf{x}(k)\big).\end{aligned} \tag{12}$$

Lemma 2: The control system (12) is said to be controlled positive if [16, 19]
$\mathbf{A}_i(\mathbf{x}(k)) + \mathbf{B}_i(\mathbf{x}(k))\mathbf{G}_j(\mathbf{x}(k)) \succcurlyeq 0$ for all $i \in \underline{p}$ and $j \in \underline{c}$.

Remark 2: In order to ease the stability analysis, system (12) is transferred to a system called dual system [19]. The equivalence of stability between the two systems under duality favours the stability analysis.

The formulation of dual system of (12) is illustrated as follows:

$$\begin{aligned}\mathbf{x}(k+1) = &\sum_{i=1}^{p}\sum_{j=1}^{p} w_i(\mathbf{x}(k))m_j(\mathbf{x}(k))\big((\mathbf{A}_i(\mathbf{x}(k))\\ &+\mathbf{B}_i(\mathbf{x}(k))\mathbf{G}_j(\mathbf{x}(k))^{\mathrm{T}}\mathbf{x}(k)\big).\end{aligned} \tag{13}$$

The proof of the equivalence of stability between two systems under duality is provided in the following.

Proof: For the non-linear discrete-time PFMB control system (12),

$$\begin{aligned}\mathbf{x}(k+1) = &\sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{x}(k))m_j(\mathbf{x}(k))\big((\mathbf{A}_i(\mathbf{x}(k))\\ &+\mathbf{B}_i(\mathbf{x}(k))\mathbf{G}_j(\mathbf{x}(k)))\mathbf{x}(k)\big)\\ = &\sum_{i_k=1}^{p}\sum_{j_k=1}^{c}\sum_{i_{k-1}=1}^{p}\sum_{j_{k-1}=1}^{c}\cdots\\ &\times\sum_{i_0=1}^{p}\sum_{j_0=1}^{c} w_i(\mathbf{x}(k))m_j(\mathbf{x}(k))w_{i_{k-1}}(\mathbf{x}(k-1))\\ &\times m_{j_{k-1}}(\mathbf{x}(k-1))\cdots w_{i_0}(\mathbf{x}(0))m_{j_0}(\mathbf{x}(0))\\ &\times\big((\mathbf{A}_i(\mathbf{x}(k))+\mathbf{B}_i(\mathbf{x}(k))\mathbf{G}_j(\mathbf{x}(k)))\\ &\times(\mathbf{A}_{i_{k-1}}(\mathbf{x}(k-1))+\mathbf{B}_{i_{k-1}}(\mathbf{x}(k-1))\\ &\times\mathbf{G}_{j_{k-1}}(\mathbf{x}(k-1)))\cdots(\mathbf{A}_{i_0}(\mathbf{x}(0))\\ &+\mathbf{B}_{i_0}(\mathbf{x}(0))\mathbf{G}_{j_0}(\mathbf{x}(0)))\mathbf{x}(0)\big).\end{aligned} \tag{14}$$

For the dual system (13), we can rewrite as

$$\begin{aligned}\mathbf{x}(k+1) = &\sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{x}(k))m_j(\mathbf{x}(k))\big((\mathbf{A}_i(\mathbf{x}(k))\\ &+\mathbf{B}_i(\mathbf{x}(k))\mathbf{G}_j(\mathbf{x}(k)))\mathbf{x}(k)\big)\\ = &\sum_{i=1}^{p}\sum_{j=1}^{c}\sum_{i_{k-1}=1}^{p}\sum_{j_{k-1}=1}^{c}\cdots\\ &\times\sum_{i_0=1}^{p}\sum_{j_0=1}^{c} w_i(\mathbf{x}(k))m_j(\mathbf{x}(k))w_{i_{k-1}}(\mathbf{x}(k-1))\\ &\times m_{j_{k-1}}(\mathbf{x}(k-1))\cdots w_{i_0}(\mathbf{x}(0))m_{j_0}(\mathbf{x}(0))\\ &\times\big((\mathbf{A}_i(\mathbf{x}(k))+\mathbf{B}_i(\mathbf{x}(k))\mathbf{G}_j(\mathbf{x}(k)))^{\mathrm{T}}\\ &\times(\mathbf{A}_{i_{k-1}}(\mathbf{x}(k-1))+\mathbf{B}_{i_{k-1}}(\mathbf{x}(k-1))\\ &\times\mathbf{G}_{j_{k-1}}(\mathbf{x}(k-1)))^{\mathrm{T}}\cdots(\mathbf{A}_{i_0}(\mathbf{x}(0))\\ &+\mathbf{B}_{i_0}(\mathbf{x}(0))\mathbf{G}_{j_0}(\mathbf{x}(0)))^{\mathrm{T}}\mathbf{x}(0)\big).\end{aligned} \tag{15}$$

Assuming the dual system (15) is stable with initial condition $\mathbf{x}(0) = \boldsymbol{\phi}(\mathbf{0}) \geq 0$, we can get $\mathbf{x}(k+1) \to 0$ as $k \to \infty$. Therefore, we can conclude that

$$\begin{aligned}&\big((\mathbf{A}_i(\mathbf{x}(k))+\mathbf{B}_i(\mathbf{x}(k))\mathbf{G}_j(\mathbf{x}(k)))^{\mathrm{T}}(\mathbf{A}_{i_{k-1}}(\mathbf{x}(k-1))\\ &\quad+\mathbf{B}_{i_{k-1}}(\mathbf{x}(k-1))\mathbf{G}_{j_{k-1}}(\mathbf{x}(k-1)))^{\mathrm{T}}\cdots(\mathbf{A}_{i_0}(\mathbf{x}(0))\\ &\quad+\mathbf{B}_{i_0}(\mathbf{x}(0))\mathbf{G}_{j_0}(\mathbf{x}(0)))^{\mathrm{T}}\big) \to 0.\end{aligned} \tag{16}$$

Since

$$\begin{aligned}&\big((\mathbf{A}_i(\mathbf{x}(k))+\mathbf{B}_i(\mathbf{x}(k))\mathbf{G}_j(\mathbf{x}(k)))^{\mathrm{T}}(\mathbf{A}_{i_{k-1}}(\mathbf{x}(k-1))\\ &\quad+\mathbf{B}_{i_{k-1}}(\mathbf{x}(k-1))\mathbf{G}_{j_{k-1}}(\mathbf{x}(k-1)))^{\mathrm{T}}\cdots(\mathbf{A}_{i_0}(\mathbf{x}(0))\\ &\quad+\mathbf{B}_{i_0}(\mathbf{x}(0))\mathbf{G}_{j_0}(\mathbf{x}(0)))^{\mathrm{T}}\big)\\ &=\big((\mathbf{A}_{i_0}(\mathbf{x}(0))+\mathbf{B}_{i_0}(\mathbf{x}(0))\mathbf{G}_{j_0}(\mathbf{x}(0)))\cdots(\mathbf{A}_{i_{k-1}}(\mathbf{x}(k-1))\\ &\quad+\mathbf{B}_{i_{k-1}}(\mathbf{x}(k-1))\mathbf{G}_{j_{k-1}}(\mathbf{x}(k-1)))(\mathbf{A}_i(\mathbf{x}(k))\\ &\quad+\mathbf{B}_i(\mathbf{x}(k))\mathbf{G}_j(\mathbf{x}(k)))\big)^{\mathrm{T}},\end{aligned} \tag{17}$$

from (16) and (17), it leads to

$$\begin{aligned}&\big((\mathbf{A}_{i_0}(\mathbf{x}(0))+\mathbf{B}_{i_0}(\mathbf{x}(0))\mathbf{G}_{j_0}(\mathbf{x}(0)))\cdots(\mathbf{A}_{i_{k-1}}(\mathbf{x}(k-1))\\ &\quad+\mathbf{B}_{i_{k-1}}(\mathbf{x}(k-1))\mathbf{G}_{j_{k-1}}(\mathbf{x}(k-1)))(\mathbf{A}_i(\mathbf{x}(k))\\ &\quad+\mathbf{B}_i(\mathbf{x}(k))\mathbf{G}_j(\mathbf{x}(k)))\big) \to 0.\end{aligned} \tag{18}$$

From (18) and (14), the stability of dual system (13) implies the stability of original system (12).

The proof of the equivalence of stability between two systems under duality is completed. □

4.1 Positivity analysis

Before conducting the stability analysis, the positivity objective which guarantees the closed loop discrete-time PFMB control systems positive, i.e. trajectory $\mathbf{x}(k) \succcurlyeq 0$ if the initial condition $\boldsymbol{\phi}(0) \succcurlyeq 0$ is realised referring to Lemma 2.

Theorem 1: The discrete-time PFMB control system (12) or its dual system (13) with the initial condition $\boldsymbol{\phi}(0) \succcurlyeq 0$ is guaranteed to be positive if there exist $\lambda_l = [\lambda_1^l, \lambda_2^l, \ldots, \lambda_n^l]^{\mathrm{T}} \succ 0$ and $\mathbf{y}_k^j(\mathbf{x}(k)) \in \Re^m$ for $l \in \underline{p}$, $j \in \underline{c}$ and $k \in \underline{n}$ such that the following SOS-based conditions are satisfied

$$a_{rs}^i(\mathbf{x}(k))\lambda_s^l + \mathbf{b}_r^i(\mathbf{x}(k))\mathbf{y}_s^{jl}(\mathbf{x}(k)) \text{ is SOS}, \; i, l \in \underline{p}; \; j \in \underline{c}, \tag{19}$$

where $a_{rs}^i(\mathbf{x}(k))$ is the (r,s)th element of the matrices $\mathbf{A}_i(\mathbf{x}(k))$; $\mathbf{B}_i(\mathbf{x}(k)) = [\mathbf{b}_1^i(\mathbf{x}(k))^{\mathrm{T}}, \mathbf{b}_2^i(\mathbf{x}(k))^{\mathrm{T}}, \ldots, \mathbf{b}_n^i(\mathbf{x}(k))^{\mathrm{T}}]^{\mathrm{T}}$, $i \in \underline{p}$, $r, s \in \underline{n}$,

$$\mathbf{G}_j(\mathbf{x}(k)) = \left[\frac{\mathbf{y}_1^{jl}(\mathbf{x}(k))}{\lambda_1^l}, \frac{\mathbf{y}_2^{jl}(\mathbf{x}(k))}{\lambda_2^l}, \cdots, \frac{\mathbf{y}_n^{jl}(\mathbf{x}(k))}{\lambda_n^l}\right] = [\mathbf{g}_1^j(\mathbf{x}(k)), \mathbf{g}_2^j(\mathbf{x}(k)), \ldots, \mathbf{g}_n^j(\mathbf{x}(k))]$$

where $\mathbf{y}_1^{jl}(\mathbf{x}(k)), \mathbf{y}_2^{jl}(\mathbf{x}(k)), \ldots, \mathbf{y}_n^{jl}(\mathbf{x}(k)) \in \mathfrak{R}^m$ for $l \in \underline{p}$ and $j \in \underline{c}$ are polynomial vectors to be determined.

4.2 Stability analysis

For the second control objective, which ensures the closed-loop discrete-time PFMB control systems asymptotically stable, i.e. trajectory $\mathbf{x}(k) \to 0$ as $k \to \infty$, the following fuzzy co-positive Lyapunov function is considered to investigate the stability of the closed-loop discrete-time PFMB control systems (12) based on Lyapunov stability theory

$$V(\mathbf{x}(k)) = \mathbf{x}^{\mathrm{T}}(k)\lambda(\mathbf{x}(k)), \quad \lambda(\mathbf{x}(k)) = \sum_{l=1}^{p} w_l(\mathbf{x}(k))\lambda_l, \tag{20}$$

where $w_l(\mathbf{x}(k)) \geq 0$ is the normalised grade of membership function corresponding to polynomial fuzzy model (3), $\lambda_l = [\lambda_1^l, \lambda_2^l, \ldots, \lambda_n^l]^{\mathrm{T}} \succ 0$.

Remark 3: As can be seen from (20), the fuzzy co-positive Lyapunov function $V(k) > 0$ is satisfied when $\lambda(\mathbf{x}(k)) \succ 0$ is ensured. Referring to the property of membership functions from (4)–(6), $\lambda(\mathbf{x}(k)) \succ 0$ can be guaranteed by defining $\lambda_l = [\lambda_1^l, \lambda_2^l, \ldots, \lambda_n^l]^{\mathrm{T}} \succ 0$.

From (20) and (13), we have

$$\begin{aligned} \triangle V(\mathbf{x}(k)) &= V(\mathbf{x}(k+1)) - V(\mathbf{x}(k)) \\ &= \mathbf{x}^{\mathrm{T}}(k+1)\lambda(\mathbf{x}(k+1)) - \mathbf{x}^{\mathrm{T}}(k)\lambda(\mathbf{x}(k)) \\ &= \mathbf{x}^{\mathrm{T}}(k)[\sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{x}(k))m_j(\mathbf{x}(k))(\mathbf{A}_i(\mathbf{x}(k)) \\ &\quad +\mathbf{B}_i(\mathbf{x}(k))\mathbf{G}_j(\mathbf{x}(k)))][\sum_{l=1}^{p} w_l(\mathbf{x}(k+1))\lambda_l] \\ &\quad -\mathbf{x}^{\mathrm{T}}(k)[\lambda(\mathbf{x}(k))]. \end{aligned} \tag{21}$$

As the following expression satisfied

$$\begin{aligned} \lambda(\mathbf{x}(k)) &= \sum_{\varsigma=1}^{p} w_\varsigma(\mathbf{x}(k))\lambda_\varsigma \\ &= \sum_{\varsigma=1}^{p}\sum_{i=1}^{p}\sum_{l=1}^{p}\sum_{j=1}^{c} w_\varsigma(\mathbf{x}(k)) \\ &\quad \times w_i(\mathbf{x}(k))w_l(\mathbf{x}(k+1))m_j(\mathbf{x}(k))\lambda_\varsigma, \end{aligned} \tag{22}$$

from (21) and (22), we have

$$\begin{aligned} \triangle V(\mathbf{x}(k)) &= \mathbf{x}^{\mathrm{T}}(k)[\sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{x}(k))m_j(\mathbf{x}(k))(\mathbf{A}_i(\mathbf{x}(k)) \\ &\quad +\mathbf{B}_i(\mathbf{x}(k))\mathbf{G}_j(\mathbf{x}(k)))][\sum_{l=1}^{p} w_l(\mathbf{x}(k+1))\lambda_l] \\ &\quad -\mathbf{x}^{\mathrm{T}}(k)[\lambda(\mathbf{x}(k))] \\ &= \mathbf{x}^{\mathrm{T}}(k)\sum_{l=1}^{p} w_l(\mathbf{x}(k+1)) \\ &\quad \times \Big[\sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{x}(k))m_j(\mathbf{x}(k))(\mathbf{A}_i(\mathbf{x}(k)) \\ &\quad +\mathbf{B}_i(\mathbf{x}(k))\mathbf{G}_j(\mathbf{x}(k)))\Big]\lambda_l - \mathbf{x}^{\mathrm{T}}(k)\Big[\sum_{\varsigma=1}^{p}\sum_{i=1}^{p} \\ &\quad \times \sum_{l=1}^{p}\sum_{j=1}^{c} w_\varsigma(\mathbf{x}(k))w_i(\mathbf{x}(k)) \\ &\quad \times w_l(\mathbf{x}(k+1))m_j(\mathbf{x}(k))\lambda_\varsigma\Big] \\ &\leq \mathbf{x}^{\mathrm{T}}(k)\sum_{l=1}^{p}\sum_{\varsigma=1}^{p}\sum_{i=1}^{p}\sum_{j=1}^{c} w_l(\mathbf{x}(k+1))w_\varsigma(\mathbf{x}(k)) \\ &\quad \times [(\mathbf{A}_i(\mathbf{x}(k)) + \mathbf{B}_i(\mathbf{x}(k))\mathbf{G}_j(\mathbf{x}(k)))\lambda_l - \lambda_\varsigma] \\ &= \mathbf{x}^{\mathrm{T}}(k)\sum_{l=1}^{p} w_l(\mathbf{x}(k+1))\sum_{\varsigma=1}^{p} w_\varsigma(\mathbf{x}(k))\mathbf{Q}_{ijl\varsigma}(\mathbf{x}(k)), \end{aligned} \tag{23}$$

where

$$\begin{aligned} \mathbf{Q}_{ijl\varsigma}(\mathbf{x}(k)) &= \sum_{i=1}^{p}\sum_{j=1}^{c} (\mathbf{A}_i(\mathbf{x}(k))\lambda_l + \mathbf{B}_i(\mathbf{x}(k)) \\ &\quad \times \sum_{s=1}^{n} \mathbf{y}_s^{jl}(\mathbf{x}(k))) - \lambda_\varsigma \\ &= \left[q_1^{ijl\varsigma}(\mathbf{x}(k)), q_2^{ijl\varsigma}(\mathbf{x}(k)), \cdots, q_n^{ijl\varsigma}(\mathbf{x}(k))\right]^{\mathrm{T}}. \end{aligned} \tag{24}$$

Remark 4: Since the stability analysis of (12) and its dual system (13) is equivalent, the PFMB control system (12) is guaranteed to be asymptotically stable, if the fuzzy controller satisfies the conditions $V(k) > 0$ and $\triangle V(\mathbf{x}(k)) < 0$ (excluding $\mathbf{x}(k) = 0$) according to Lyapunov stability theory. This can be achieved by $\mathbf{Q}_{ijl\varsigma}(\mathbf{x}(k)) \prec 0$ for all i, l and $\varsigma \in \underline{p}$ and $j \in \underline{c}$.

The stability analysis result is summarised in the following theorem.

Theorem 2: The discrete-time PFMB control system (12) is positive and asymptotically stable if there exist $\lambda(\mathbf{x}(k)) = \sum_{l=1}^{p} w_l(\mathbf{x}(k))\lambda_l \in \mathfrak{R}^n$ and $\mathbf{y}_s^{jl}(\mathbf{x}(k)) \in \mathfrak{R}^m$ for $l \in \underline{p}, j \in \underline{c}, s \in \underline{n}$ such that Theorem 1 and the following SOS-based conditions are satisfied

$$\lambda_s^l - \varepsilon_1 \text{ is SOS}, \quad l \in \underline{p}, s \in \underline{n}, \tag{25}$$

$$-q_s^{ijl\varsigma}(\mathbf{x}(k)) - \varepsilon_2(\mathbf{x}(k)) \text{ is SOS}, \quad i, l, \varsigma \in \underline{p}, j \in \underline{c}, s \in \underline{n}, \tag{26}$$

where $\varepsilon_1 > 0$ and $\varepsilon_2(\mathbf{x}(k)) > 0$ are predefined scalar and polynomial, respectively, $q_s^{ijl\varsigma}(\mathbf{x}(k))$ is defined in (24), and the feedback gains and the other variables are defined in Theorem 1. The decision variable λ_l is obtained by the SOS formulation satisfying Theorems 1 and 2. In order to impose the constraint $x_k \geq 0$, the technique of variable transformation is employed which simply turns $x_s(k)$ to $x_s(k)^2$, $s \in \underline{n}$.

4.3 Stability and positivity analysis with bounded control signals

The third control objective is to ensures control signals of the closed-loop discrete-time PFMB control systems asymptotically bounded with certain range, i.e. trajectory $0 \preceq \mathbf{u}(k) \prec \bar{\mathbf{u}}$ as $k \in [0 \quad \infty]$, where $\bar{\mathbf{u}} \succ 0$ is a predefined positive constant vector which used to denotes the bounds as the function of limiting the value of control signals.

The SOS-based bounds condition is summarised in the following theorem.

Theorem 3: The discrete-time PFMB control system (12) is positive and asymptotically stable with bounded control if there exist $\bar{\mathbf{u}} \succ 0$, $\mathbf{y}_s^{jl}(\mathbf{x}(k)) \in \Re^m$ for $l \in \underline{p}$ $j \in \underline{c}$ and $s \in \underline{n}$ and initial condition $0 \preceq \boldsymbol{\phi}(0) \preceq \lambda_l$ such that Theorems 1 and 2 and the following SOS-based conditions are satisfied

$$\mathbf{y}_s^{jl}(\mathbf{x}(k)) - \varepsilon_3(\mathbf{x}(k)) \text{ is SOS}, \quad l \in \underline{p}, j \in \underline{c}, s \in \underline{n}, \tag{27}$$

$$-\sum_{j=1}^{c}\sum_{s=1}^{n}\mathbf{y}_s^{jl}(\mathbf{x}(k)) + \bar{\mathbf{u}} - \varepsilon_4(\mathbf{x}(k)) \text{ is SOS}, \quad l \in \underline{p}, j \in \underline{c}, s \in \underline{n}, \tag{28}$$

$$\lambda_k = \bar{\mathbf{x}}, \tag{29}$$

where λ_k is represented as the minimum $\lambda_l, l \in \underline{p}$ and $\varepsilon_3(\mathbf{x}(k)) > 0$ and $\varepsilon_4(\mathbf{x}(k)) > 0$ are predefined $\Re^m$ polynomials vector, $\bar{\mathbf{u}} \succ 0$ is a predefined $\Re^m$ positive constant vector. $\bar{\mathbf{x}}$ is predefined $\Re^n$ constant vector which represents upper boundary information of operating domain.

Proof: First we prove the state feedback polynomial fuzzy controller
$\mathbf{u}(k) = \sum_{j=1}^{c} m_j(\mathbf{x}(k))\mathbf{G}_j(\mathbf{x}(k))\mathbf{x}(k) \succeq 0$.

In this paper, we need to design the feedback gains of polynomial fuzzy controller $\mathbf{G}_j(\mathbf{x}(k))$ such that the state feedback polynomial fuzzy controller $\mathbf{u}(k) = \sum_{j=1}^{c} m_j(\mathbf{x}(k))\mathbf{G}_j(\mathbf{x}(k))\mathbf{x}(k)$ satisfying $0 \preceq \mathbf{u}(k) = \sum_{j=1}^{c} m_j(\mathbf{x}(k))\mathbf{G}_j(\mathbf{x}(k))\mathbf{x}(k)$. From the first SOS-based condition (27) in Theorem 3 we can get $\mathbf{y}_s^{jl}(\mathbf{x}(k)) \succ 0$ for $l \in \underline{p}, j \in \underline{c}, s \in \underline{n}$, simultaneously, referring to Remark 2, we can get $\lambda_s^l > 0$ for $l \in \underline{p}, s \in \underline{n}$. Since the formulation of polynomial fuzzy controller

$$\mathbf{G}_j(\mathbf{x}(k)) = \left[\frac{\mathbf{y}_1^{jl}(\mathbf{x}(k))}{\lambda_1^l}, \frac{\mathbf{y}_2^{jl}(\mathbf{x}(k))}{\lambda_2^l}, \ldots, \frac{\mathbf{y}_n^{jl}(\mathbf{x}(k))}{\lambda_n^l}\right] = [\mathbf{g}_1^j(\mathbf{x}(k)), \mathbf{g}_2^j(\mathbf{x}(k)), \ldots, \mathbf{g}_n^j(\mathbf{x}(k))],$$

we can get $\mathbf{G}_j(\mathbf{x}(k)) \succ 0$. Furthermore, the SOS-based positivity conditions in Theorem 1 guarantee $\mathbf{x}(k) \succeq 0$, therefore we can obtain $\mathbf{u}(k) = \sum_{j=1}^{c} m_j(\mathbf{x}(k))\mathbf{G}_j(\mathbf{x}(k))\mathbf{x}(k) \succeq 0$.

In the following we prove the state feedback polynomial fuzzy controller
$\mathbf{u}(k) = \sum_{j=1}^{c} m_j(\mathbf{x}(k))\mathbf{G}_j(\mathbf{x}(k))\mathbf{x}(k) \prec \bar{\mathbf{u}}$. From the Theorem 3, we get the initial condition $0 \preceq \boldsymbol{\phi}(0) \preceq \lambda_l$, therefore we can get $\mathbf{u}(k) = \sum_{j=1}^{c} m_j(\mathbf{x}(k))\mathbf{G}_j(\mathbf{x}(k))\mathbf{x}(k) \preceq \sum_{j=1}^{c} m_j(\mathbf{x}(k))\mathbf{G}_j(\mathbf{x}(k))\lambda_l$ for any initial condition $0 \preceq \boldsymbol{\phi}(0) \preceq \lambda_l$ and $l \in \underline{p}$. If we show that $\mathbf{x}(k) \prec \lambda_l$, since the SOS-based positivity condition (19) in Theorem 1 implies $\mathbf{A}_i(\mathbf{x}(k)) + \mathbf{B}_i(\mathbf{x}(k))\mathbf{G}_j(\mathbf{x}(k)) \succeq 0$ and from polynomial fuzzy model (12) we can have

$$\begin{aligned}\mathbf{x}(k+1) &= \sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{x}(k))m_j(\mathbf{x}(k))((\mathbf{A}_i(\mathbf{x}(k)) \\ &\quad + \mathbf{B}_i(\mathbf{x}(k))\mathbf{G}_j(\mathbf{x}(k)))\mathbf{x}(k)) \\ &\preceq \sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{x}(k))m_j(\mathbf{x}(k))(\mathbf{A}_i(\mathbf{x}(k))\lambda_l \\ &\quad + \mathbf{B}_i(\mathbf{x}(k))\sum_{s=1}^{n}\mathbf{y}_s^{jl}(\mathbf{x}(k)))\,.\end{aligned} \tag{30}$$

Furthermore, from the second SOS-based stability condition (26) implies $q_s^{ijl\varsigma}(\mathbf{x}(k)) < 0$ for $i, l, \varsigma \in \underline{p}, j \in \underline{c}, s \in \underline{n}$, therefore we have

$$\begin{aligned}\mathbf{Q}_{ijl\varsigma}(\mathbf{x}(k)) = \sum_{i=1}^{p}\sum_{j=1}^{c}(\mathbf{A}_i(\mathbf{x}(k))\lambda_l + \mathbf{B}_i(\mathbf{x}(k)) \\ \times \sum_{s=1}^{n}\mathbf{y}_s^{jl}(\mathbf{x}(k))) - \lambda_\varsigma \prec 0.\end{aligned} \tag{31}$$

From (30) and (31), we can get

$$\begin{aligned}\mathbf{x}(k+1) &\preceq \sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{x}(k))m_j(\mathbf{x}(k))(\mathbf{A}_i(\mathbf{x}(k))\lambda_l \\ &\quad + \mathbf{B}_i(\mathbf{x}(k))\sum_{s=1}^{n}\mathbf{y}_s^{jl}(\mathbf{x}(k))) \\ &\prec \sum_{i=1}^{p}\sum_{j=1}^{c}(\mathbf{A}_i(\mathbf{x}(k))\lambda_l + \mathbf{B}_i(\mathbf{x}(k))\sum_{s=1}^{n}\mathbf{y}_s^{jl}(\mathbf{x}(k))) \\ &\prec \lambda_\varsigma\,.\end{aligned} \tag{32}$$

where i, l and $\varsigma \in \underline{p}$ and $j \in \underline{c}$. Hence

$$\begin{aligned}\mathbf{x}(k+1) &\prec \sum_{i=1}^{p}\sum_{j=1}^{c}(\mathbf{A}_i(\mathbf{x}(k))\lambda_l + \mathbf{B}_i(\mathbf{x}(k))\sum_{s=1}^{n}\mathbf{y}_s^{jl}(\mathbf{x}(k))) \\ &\prec \lambda_l, \quad \text{for all } l \in \underline{p}\,.\end{aligned} \tag{33}$$

This proof process can apply to prove $\mathbf{x}(k) \prec \lambda_l$ if initial condition $0 \preceq \boldsymbol{\phi}(0) \preceq \lambda_l$. Therefore,

$$\begin{aligned}\mathbf{u}(k) &= \sum_{j=1}^{c} m_j(\mathbf{x}(k))\mathbf{G}_j(\mathbf{x}(k))\mathbf{x}(k) \\ &\preceq \sum_{j=1}^{c}\mathbf{G}_j(\mathbf{x}(k))\mathbf{x}(k) \\ &\prec \sum_{j=1}^{c}\mathbf{G}_j(\mathbf{x}(k))\lambda_l \\ &= \sum_{j=1}^{c}\sum_{s=1}^{n}\mathbf{y}_s^{jl}(\mathbf{x}(k))\,.\end{aligned} \tag{34}$$

Furthermore, the second SOS-based bounded condition (28) implies $\sum_{j=1}^{c}\sum_{s=1}^{n}\mathbf{y}_s^{jl}(\mathbf{x}(k)) \prec \bar{\mathbf{u}}$ for $l \in \underline{p}, j \in \underline{c}$ and $s \in \underline{n}$, therefore we have $\mathbf{u}(k) = \sum_{j=1}^{c} m_j(\mathbf{x}(k))\mathbf{G}_j(\mathbf{x}(k))\mathbf{x}(k) \prec \bar{\mathbf{u}}$.

Additionally, the third SOS-based bounded condition (29) implies the minimum $\lambda_l, l \in \underline{p}$ which represents that λ_k satisfies $\lambda_k = \bar{\mathbf{x}}$, therefore we can guarantee system states $\mathbf{x}(k) \preceq \bar{\mathbf{x}}$ which ensures positive PFMB control system can always work in the whole operating domain.

The proof is completed. □

Remark 5: To demonstrate that the fuzzy co-positive Lyapunov function can produce more relaxed positivity, stability and bounded control conditions compared with common co-positive Lyapunov function because of its generality, now we introduce common co-positive Lyapunov function as following

$$V(\mathbf{x}(k)) = \mathbf{x}^{\mathrm{T}}(k)\lambda, \tag{35}$$

From (13) and (35), we have

$$\begin{aligned}\triangle V(\mathbf{x}(k)) &= V(\mathbf{x}(k+1)) - V(\mathbf{x}(k))\\ &= \sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{x}(k))m_j(\mathbf{x}(k))[\mathbf{x}^{\mathrm{T}}(k)(\mathbf{A}_i(\mathbf{x}(k))\\ &\quad +\mathbf{B}_i(\mathbf{x}(k))\mathbf{G}_j(\mathbf{x}(k)))]\lambda\\ &= \sum_{i=1}^{p}\sum_{j=1}^{c} w_i(\mathbf{x}(k))m_j(\mathbf{x}(k))\mathbf{x}^{\mathrm{T}}(k)[\mathbf{A}_i(\mathbf{x}(k))\lambda \\ &\quad +\mathbf{B}_i(\mathbf{x}(k))\sum_{s=1}^{n}\mathbf{y}_s^j(\mathbf{x}(k)) - \lambda]\\ &\le \sum_{i=1}^{p}\sum_{j=1}^{c}\mathbf{x}^{\mathrm{T}}(k)\mathbf{Q}_{ij}(\mathbf{x}(k))\end{aligned} \quad (36)$$

where

$$\mathbf{Q}_{ij}(\mathbf{x}(k)) = \mathbf{A}_i(\mathbf{x}(k))\lambda + \mathbf{B}_i(\mathbf{x}(k))\sum_{s=1}^{n}\mathbf{y}_s^j(\mathbf{x}(k)) - \lambda\,. \quad (37)$$

and

$$\mathbf{G}_j(\mathbf{x}(k)) = \left[\frac{\mathbf{y}_1^j(\mathbf{x}(k))}{\lambda_1}, \frac{\mathbf{y}_2^j(\mathbf{x}(k))}{\lambda_2}, \cdots, \frac{\mathbf{y}_n^j(\mathbf{x}(k))}{\lambda_n}\right],$$

$\mathbf{y}_1^j(\mathbf{x}(k)), \mathbf{y}_2^j(\mathbf{x}(k)), \ldots, \mathbf{y}_n^j(\mathbf{x}(k)) \in \Re^m$ for $j \in \underline{c}$ are to be determined.

Referring to SOS-based conditions from Theorem 3 for fuzzy co-positive Lyapunov function, the positivity condition $\mathbf{A}_i(\mathbf{x}(k))\lambda + \mathbf{B}_i(\mathbf{x}(k))\sum_{s=1}^{n}\mathbf{y}_s^j(\mathbf{x}(k)) \ge 0$, stability conditions $\lambda \succ 0$ and $\mathbf{Q}_{ij}(\mathbf{x}(k)) \prec 0$ and bounded control conditions $\mathbf{y}_s^j(\mathbf{x}(k)) \succ 0$, $\sum_{j=1}^{c}\sum_{s=1}^{n}\mathbf{y}_s^j(\mathbf{x}(k)) - \bar{\mathbf{u}} \prec 0$ and $\lambda = \bar{\mathbf{x}}$ for common co-positive Lyapunov function guarantee the positivity, stability and bounded control for discrete-time PFMB control system (12) and its dual system (13).

Compared the formulation of common co-positive Lyapunov function (35) and fuzzy co-positive Lyapunov function (20), we can get constant vector λ in (35) is replaced by $\lambda(\mathbf{x}(k))$ in (20) which lead to relaxed feasible solution of positivity, stability and bounded control conditions for solver to find attainably. Therefore, the conservativeness of positivity, stability and bounded control conditions can be reduced to a certain extent.

5 Simulation example

In this section, the proposed theorem is validated using a simulation example. A discrete-time polynomial fuzzy model with three rules is considered with the following sub-systems and input matrices

$$\begin{aligned}&\mathbf{x}(k) = [x_1(k) \quad x_2(k)]^{\mathrm{T}},\\ &\mathbf{A}_{10}(x_1(k))\\ &\quad = \begin{bmatrix} 0.06b + 0.7 + 0.015x_1(k) - 0.001x_1(k)^2 & 0.2\\ 0.2 & 0.3\end{bmatrix},\\ &\mathbf{A}_{20}(x_1(k)) = \begin{bmatrix} 0.4 & 0.1 - 0.01x_1(k)\\ 0.2 & 0.1a\end{bmatrix},\end{aligned}$$

$$\begin{aligned}&\mathbf{A}_{30}(x_1(k)) = \begin{bmatrix} 0.03 & 0.4\\ 0.24 + 0.01x_1(k) & 0.06 + 0.0003x_1(k)^2\end{bmatrix},\\ &\mathbf{B}_1(x_1(k)) = \begin{bmatrix} -0.3b + 0.4\\ 0.1 - 0.001x_1(k)^2\end{bmatrix},\\ &\mathbf{B}_2(x_1(k)) = \begin{bmatrix} 1 + 0.015x_1(k)^2\\ -0.1 + 0.001x_1(k)^2\end{bmatrix},\\ &\mathbf{B}_3(x_1(k)) = \begin{bmatrix} -1 + 0.005x_1(k)^2\\ 0.1 - 0.001x_1(k)^2\end{bmatrix},\\ &\bar{\mathbf{u}} = 2.\end{aligned}$$

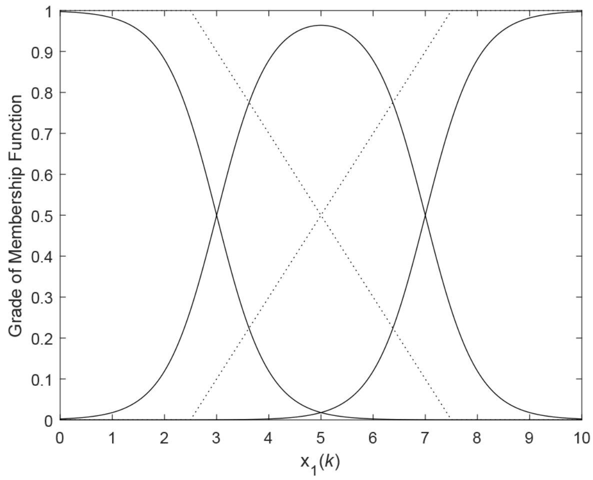

Fig. 1 *The fuzzy model membership functions (solid lines) and fuzzy controller membership functions (dotted lines)*

The parameters a and b are constant parameters chosen in the range of $1.5 \le a \le 5$ and $5.3 \le b \le 6.1$ at the interval of 0.5 and 0.2, respectively.

The membership functions of three-rule polynomial fuzzy model are chosen as

$$w_1(x_1(k)) = 1 - \frac{1}{(1 + e^{-(2x_1(k) - 6)})},$$

$$w_2(x_1(k)) = 1 - w_1(x_1(k)) - w_3(x_1(k)), \quad w_3(x_1(k)) = \frac{1}{(1 + e^{-(2x_1(k) - 14)})}.$$

Under imperfect premise matching design concept, we choose a two-rule polynomial fuzzy controller to guarantee the positivity and stability of the system and bounded control signal. The membership functions of the fuzzy controller are chosen as follows:

$$m_1(x_1(k)) = \begin{cases} 1, & \text{for } x_1(k) < 2.5\\ \dfrac{-x_1(k) + 7.5}{5}, & \text{for } 2.5 \le x_1(k) \le 7.5\\ 0, & \text{for } x_1(k) > 7.5\end{cases}$$

and $m_2(x_1(k)) = 1 - m_1(x_1(k))$. Fig. 1 shows the shape of membership functions. The control objective is that the PFMB control system is guaranteed to be asymptotically stable, positive and bounded control in the operating domain $x_1(k) \in [0 \quad 10], x_2(k) \in [0 \quad 15]$.

Based on the SOS-based positivity, stability and bounded control conditions derived from Theorem 3, we set $\varepsilon_1 = \varepsilon_2(\mathbf{x}(k)) = \varepsilon_3(\mathbf{x}(k)) = \varepsilon_4(\mathbf{x}(k)) = 0.0010$, and $\mathbf{y}_s^{jl}(\mathbf{x}(k))$ for $l \in \underline{p}$ $j \in \underline{c}$ and $s \in \underline{n}$ as polynomial of degree 0–4 in $x_1(k)$. The results for fuzzy co-positive Lyapunov function are shown in Fig. 2 where the feasible regions satisfying SOS-based positivity, stability and bounded control conditions are shown with '∘'. Based on the conditions referring to Remark 5, we set $\mathbf{y}_s^j(\mathbf{x}(k))$ for $j \in \underline{c}$ and $s \in \underline{n}$ as polynomial of degree 0 to 4 in $x_1(k)$. The results for common co-positive Lyapunov function are shown in Fig. 2 where the feasible regions satisfying SOS-based positivity, stability and bounded control conditions are shown with '×'.

It is visible in Fig. 2 that the feasible regions obtained based on fuzzy co-positive Lyapunov function are larger than those are obtained based on common co-positive Lyapunov function. Therefore, we can get the fuzzy co-positive Lyapunov function provides more relaxed results than common co-positive Lyapunov function.

To verify the effectiveness of the proposed method, first, the transient response of system states $\mathbf{x}(k)$ with initial conditions

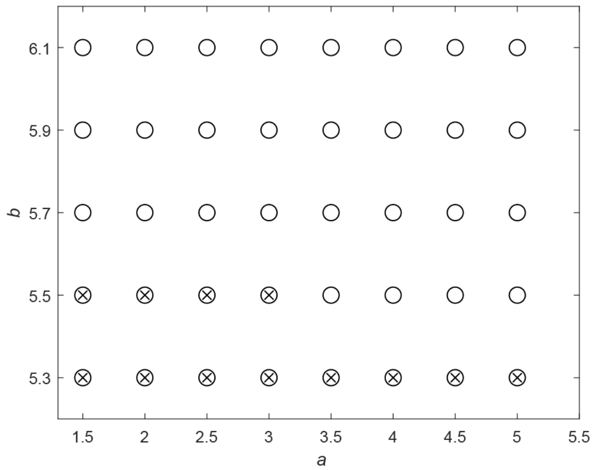

Fig. 2 *The feasible regions given by common co-positive Lyapunov function indicated by '×' based on Remark 5, given by fuzzy co-positive Lyapunov function indicated by '∘' based on Theorem 3*

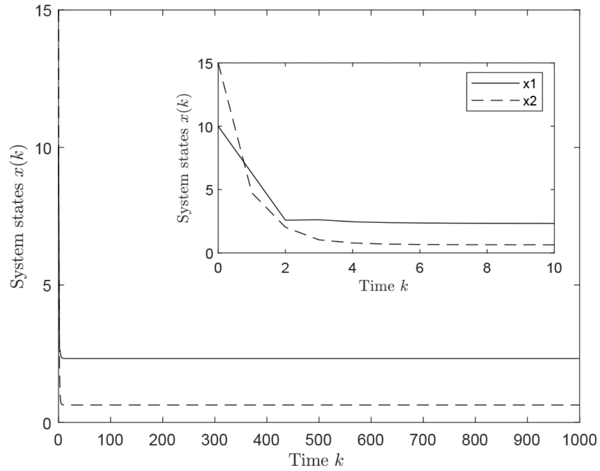

Fig. 3 *Transient response of open-loop system states* $\mathbf{x}(k)$ *with the initial condition* $\phi(0) = [10, 15]^T$ *for* $a = 1.5, b = 6.1$

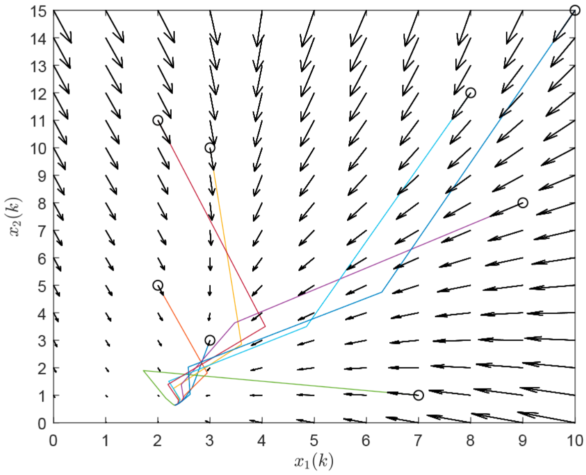

Fig. 4 *Open-loop phase plot of* $x_1(k)$ *and* $x_2(k)$ *for* $a = 1.5, b = 6.1$

$\phi(0) = [10, 15]^T$ for $a = 1.5, b = 6.1$ is conducted (see Fig. 3). The open-loop phase plot of system states $x_1(k)$ and $x_2(k)$ for $a = 1.5, b = 6.1$ with eight different initial conditions indicated by '∘' including $\phi(0) = [3, 3]^T, [2, 5]^T, [3, 10]^T, [9, 8]^T, [7, 1]^T, [8, 12]^T,$ $[2, 11]^T, [10, 15]^T [2, 11]^T, [10, 15]^T$ are conducted (see Fig. 4). As can be seen from Figs. 3 to 4, the original open-loop discrete time system is unstable. Referring to the obtained feasible regions based on fuzzy co-positive Lyapunov function, $a = 1.5, b = 6.1$ indicated by '∘' in Fig. 2 based on Theorem 3 can be employed to

Table 1 Feedback gains of polynomial fuzzy controller corresponding to the feasible region indicated by the symbols '∘' for $a = 1.5, b = 6.1$ referring to Fig. 2 based on Theorem 3

$\mathbf{G}_j(\mathbf{x}(k))$	Parameters for polynomial fuzzy controller
$\mathbf{G}_1(\mathbf{x}(k))$	$0.3613 \times 10^{-5} x_1^4 - 0.2441 \times 10^{-3} x_1^3$ $+0.6100 \times 10^{-2} x_1^2 - 0.6402 \times 10^{-1} x_1$ $+0.2384,$ $0.2214 \times 10^{-5} x_1^4 - 0.1489 \times 10^{-3} x_1^3$ $+0.3630 \times 10^{-2} x_1^2 - 0.3770 \times 10^{-1} x_1$ $+0.1551,$
$\mathbf{G}_2(\mathbf{x}(k))$	$0.3613 \times 10^{-5} x_1^4 - 0.2441 \times 10^{-3} x_1^3$ $+0.6100 \times 10^{-2} x_1^2 - 0.6402 \times 10^{-1} x_1$ $+0.2384,$ $0.2214 \times 10^{-5} x_1^4 - 0.1489 \times 10^{-3} x_1^3$ $+0.3630 \times 10^{-2} x_1^2 - 0.3770 \times 10^{-1} x_1$ $+0.1551.$

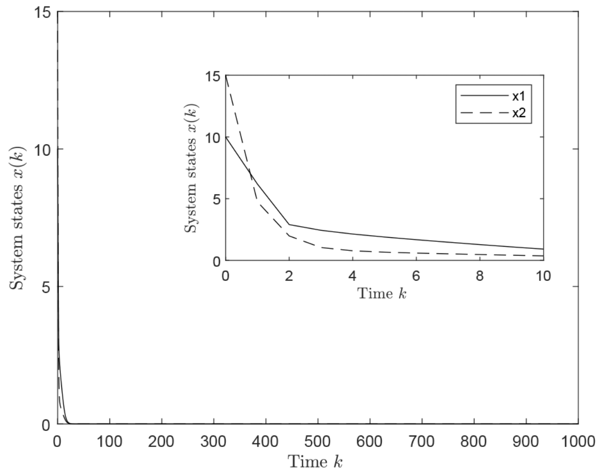

Fig. 5 *Transient response of system states* $\mathbf{x}(k)$ *with the initial condition* $\phi(0) = [10, 15]^T$ *(the feasible region indicated by the symbol '∘' based on Theorem 3 for* $a = 1.5, b = 6.1$ *referring to Fig. 2)*

guarantee the stability and positivity of discrete time polynomial based control system with bounded control. In order to verify the result, the transient response of system states, control signal and feedback gains of fuzzy controller and phase plot of system states are conducted together. $\lambda_1 = [10, 15]^T$, $\lambda_2 = [26.19, 39.29]^T$ and $\lambda_3 = [34.19, 51.29]^T$ are obtained and feedback gains of polynomial fuzzy controller are obtained in Table 1.

The transient response of system states $\mathbf{x}(k)$, control signal $\mathbf{u}(k)$ and feedback gains $\mathbf{G}_j(k)$ with initial conditions $\phi(0) = [10, 15]^T$ (see Figs. 5–7) are illustrated that the polynomial fuzzy controller is always able to guarantee the positivity, stability and bounded control for non-linear polynomial fuzzy control system when $0 \le \boldsymbol{\phi}(0) \le \lambda_l$ for all $l \in \underline{p}$.

As can be seen from Fig. 5, confined with range of $\mathbf{0} \le \boldsymbol{\phi}(0) \le \lambda_l$ for all $l \in \underline{p}$, the polynomial fuzzy controller in Table 1 obtained based on Theorem 3 can guarantee the system positivity and stability. As in Fig. 6, the polynomial fuzzy controller in Table 1 can guarantee the bounded control signal $0 \le \mathbf{u}(k) < 2$. Also in Fig. 7, the polynomial fuzzy controller obtained in Table 1 can guarantee the value of feedback gains of polynomial fuzzy controller $\mathbf{G}_j(k) \succ 0$. Therefore, the control objectives including positivity, stability and bounded control are all realised.

To ensure the validity of results, the phase plots of $x_1(k)$ and $x_2(k)$ are simulated with eight different initial conditions indicated by '∘' including

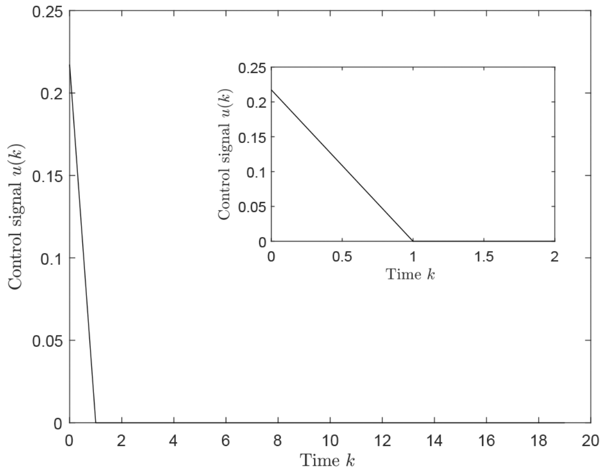

Fig. 6 *Control signal* $\mathbf{u}(k)$ *with the initial condition* $\phi(0) = [10, 15]^T$ *(the feasible region indicated by the symbol '∘' based on Theorem 3 for* $a = 1.5, b = 6.1$ *referring to Fig. 2)*

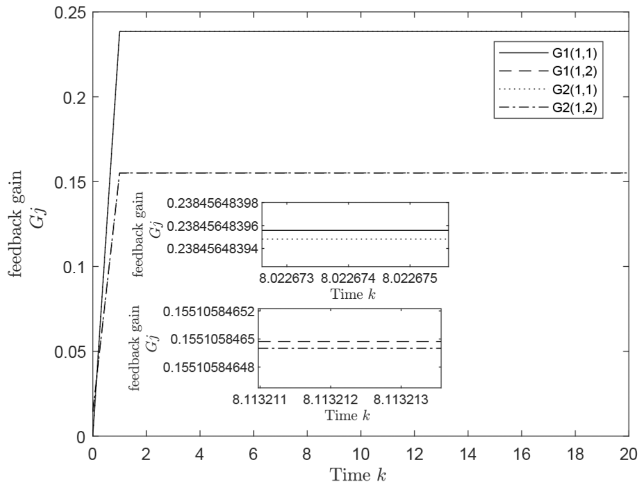

Fig. 7 *Feedback gains of fuzzy controller* $\mathbf{G}_j(k)$ *with the initial condition* $\phi(0) = [10, 15]^T$ *(the feasible region indicated by the symbol '∘' based on Theorem 3 for* $a = 1.5, b = 6.1$ *referring to Fig. 2)*

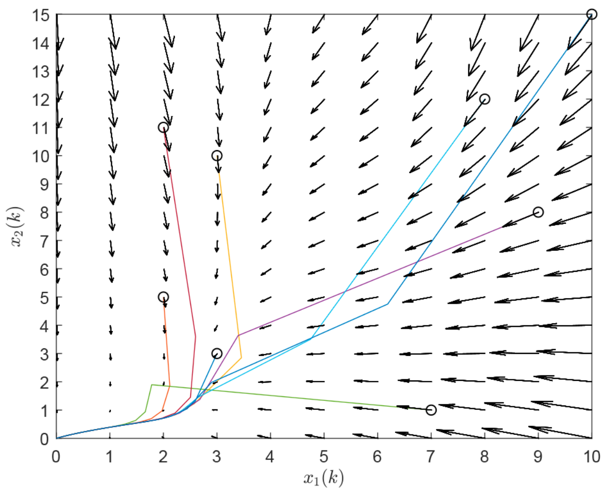

Fig. 8 *Phase plot of* $x_1(k)$ *and* $x_2(k)$ *(the feasible region indicated by the symbol '∘' based on Theorem 3 for* $a = 1.5., b = 6.1$ *referring to Fig. 2)*

$$\phi(0) = [3, 3]^{\mathrm{T}}, [2, 5]^{\mathrm{T}}, [3, 10]^{\mathrm{T}}, [9, 8]^{\mathrm{T}}, [7, 1]^{\mathrm{T}}, [8, 12]^{\mathrm{T}}, [2, 11]^{\mathrm{T}}, [10, 15]^{\mathrm{T}}[2, 11]^{\mathrm{T}}, [10, 15]^{\mathrm{T}} \tag{38}$$

The results (see Fig. 8) show that the polynomial fuzzy controller is able to drive all the system states to equilibrium

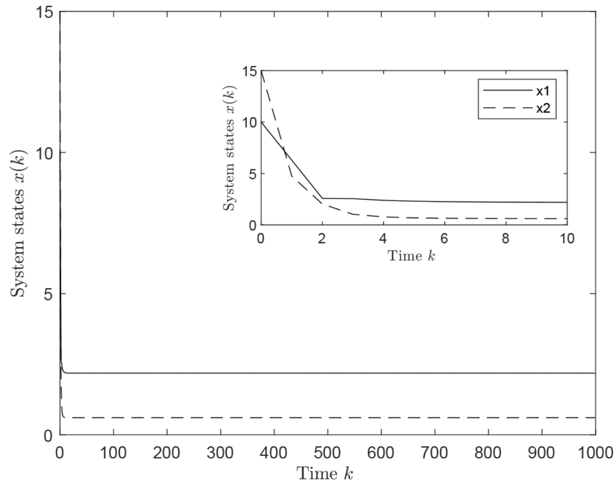

Fig. 9 *Transient response of open-loop system states* $\mathbf{x}(k)$ *with the initial condition* $\phi(0) = [10, 15]^T$ *for* $a = 1.5, b = 5.5$

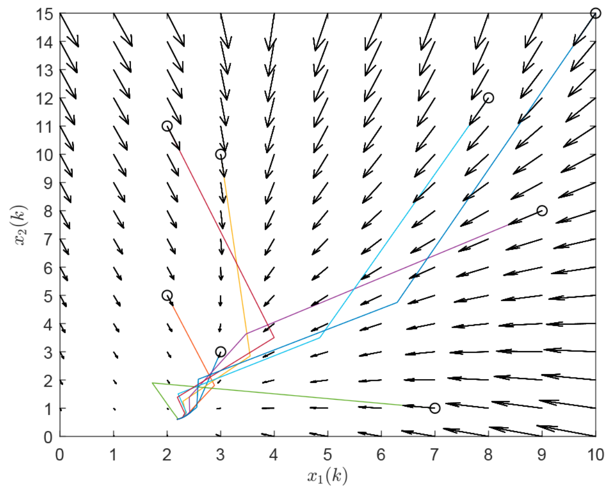

Fig. 10 *Open-loop phase plot of* $x_1(k)$ *and* $x_2(k)$ *for* $a = 1.5, b = 5.5$

(origin) while always hold them positive based on different initial conditions.

In the next, the transient response of system states $\mathbf{x}(k)$ with initial conditions $\phi(0) = [10, 15]^{\mathrm{T}}$ for $a = 1.5, b = 5.5$ are conducted (see Fig. 9). The open-loop phase plot of system states $x_1(k)$ and $x_2(k)$ for $a = 1.5, b = 6.1$ with 8 different initial conditions indicated by '∘' including

$$\phi(0) = [3, 3]^{\mathrm{T}}, [2, 5]^{\mathrm{T}}, [3, 10]^{\mathrm{T}}, [9, 8]^{\mathrm{T}}, [7, 1]^{\mathrm{T}}, [8, 12]^{\mathrm{T}}, [2, 11]^{\mathrm{T}}, [10, 15]^{\mathrm{T}}, [2, 11]^{\mathrm{T}}, [10, 15]^{\mathrm{T}} \tag{39}$$

are conducted (see Fig. 10). As can be seen from Figs. 9 to 10, the original open-loop discrete time system is unstable. Referring to the obtained feasible regions based on common co-positive Lyapunov function, $a = 1.5, b = 5.5$ indicated by '×' in Fig. 2 based on Remark 5 can be employed to guarantee the stability and positivity of discrete time polynomial based control system with bounded control. In order to verify the result, the transient response of system states, control signal and feedback gains of fuzzy controller and phase plot of system states are conducted together. $\lambda = [10, 15]^{\mathrm{T}}$ is obtained and feedback gains of polynomial fuzzy controller are obtained in Table 2.

The transient response of system states $\mathbf{x}(k)$, control signal $\mathbf{u}(k)$ and feedback gains $\mathbf{G}_j(k)$ with initial conditions $\phi(0) = [10, 15]^{\mathrm{T}}$ (see Figs. 11–13) are illustrated that the polynomial fuzzy controller is always able to guarantee the positivity, stability and bounded control for non-linear polynomial fuzzy control system when $0 \preceq \boldsymbol{\phi}(0) \preceq \lambda$.

Table 2 Feedback gains of polynomial fuzzy controller corresponding to the feasible region indicated by the symbols '×' for $a = 1.5, b = 5.5$ referring to Fig. 2 based on Remark 5

$\mathbf{G}_j(\mathbf{x}(k))$	Parameters for polynomial fuzzy controller
$\mathbf{G}_1(\mathbf{x}(k))$	$0.1755\times10^{-6}x_1^4 - 0.4967\times10^{-5}x_1^3 + 0.1325\times10^{-3}x_1^2 - 0.2904\times10^{-2}x_1 + 0.6671\times10^{-1}$, $0.1170\times10^{-6}x_1^4 - 0.3311\times10^{-5}x_1^3 + 0.8834\times10^{-4}x_1^2 - 0.1936\times10^{-2}x_1 + 0.4447\times10^{-1}$
$\mathbf{G}_2(\mathbf{x}(k))$	$0.1755\times10^{-6}x_1^4 - 0.4967\times10^{-5}x_1^3 + 0.1325\times10^{-3}x_1^2 - 0.2904\times10^{-2}x_1 + 0.6671\times10^{-1}$, $0.1170\times10^{-6}x_1^4 - 0.3311\times10^{-5}x_1^3 + 0.8834\times10^{-4}x_1^2 - 0.1936\times10^{-2}x_1 + 0.4447\times10^{-1}$

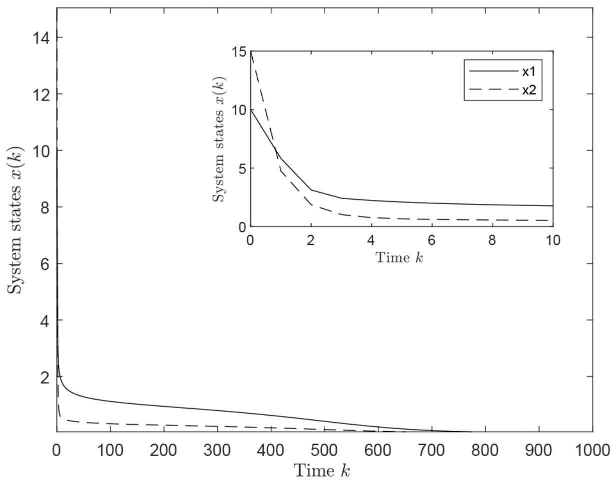

Fig. 11 *Transient response of system states* $\mathbf{x}(k)$ *with the initial condition* $\phi(0) = [10, 15]^T$ *(the feasible region indicated by the symbol '×' based on Remark 5 for* $a = 1.5, b = 5.5$ *referring to Fig. 2)*

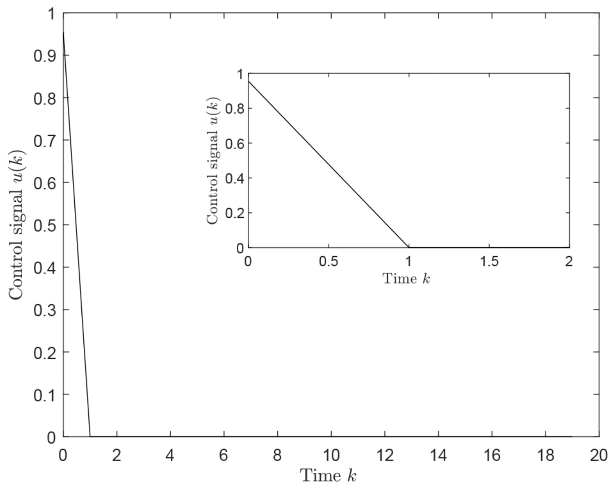

Fig. 12 *Control signal* $\mathbf{u}(k)$ *with initial condition* $\phi(0) = [10, 15]^T$ *(the feasible region indicated by the symbol '×' based on Remark 5 for* $a = 1.5, b = 5.5$ *referring to Fig. 2)*

As seen in Fig. 11, considering $\mathbf{0} \preceq \phi(0) \preceq \lambda$, the polynomial fuzzy controller obtained in Table 2 can guarantee the system positivity and stability. As seen in Fig. 12, the polynomial fuzzy controller obtained in Table 2 guarantees the bounded control

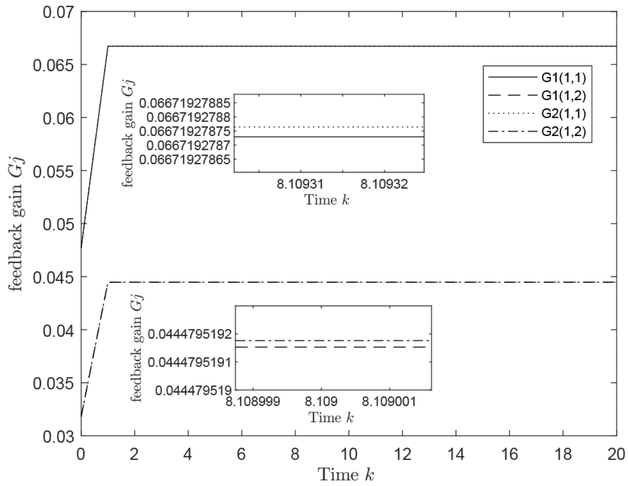

Fig. 13 *Feedback gains of fuzzy controller* $\mathbf{G}_j(k)$ *with the initial condition* $\phi(0) = [10, 15]^T$ *(the feasible region indicated by the symbol '×' based on Remark 5 for* $a = 1.5, b = 5.5$ *referring to Fig. 2)*

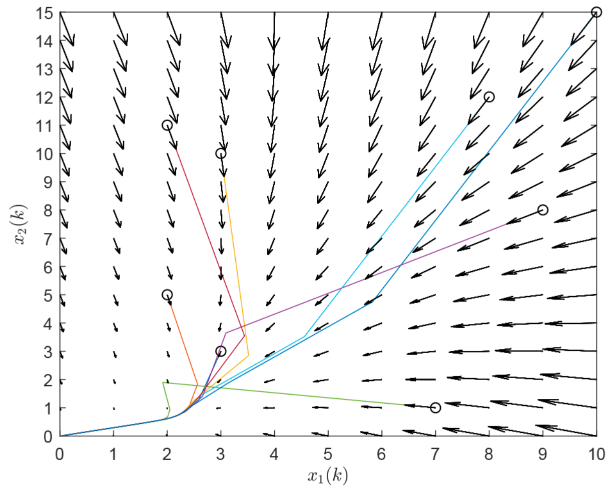

Fig. 14 *Phase plot of* $x_1(k)$ *and* $x_2(k)$ *(the feasible region indicated by the symbol '×' based on Remark 5 for* $a = 1.5, b = 5.5$ *referring to Fig. 2)*

signal $0 \le \mathbf{u}(k) < 2$ and the value of feedback gains of polynomial fuzzy controller $\mathbf{G}_j(k) \succ 0$ (see Fig. 13). Therefore, the control objectives including positivity, stability and bounded control are all realised.

To ensure the validity of results, the phase plots of $x_1(k)$ and $x_2(k)$ are simulated with eight different initial conditions indicated by '∘' including

$$\begin{aligned}\phi(0) = {} & [3,3]^{\mathrm{T}}, [2,5]^{\mathrm{T}}, [3,10]^{\mathrm{T}}, [9,8]^{\mathrm{T}}, [7,1]^{\mathrm{T}}, [8,12]^{\mathrm{T}}, \\ & [2,11]^{\mathrm{T}}, [10,15]^{\mathrm{T}}, [2,11]^{\mathrm{T}}, [10,15]^{\mathrm{T}}\end{aligned} \tag{40}$$

.

The results (see Fig. 14) show that the polynomial fuzzy controller is able to drive all the system states to equilibrium (origin) while always hold them positive based on different initial conditions.

From the simulation results, we can get fuzzy co-positive Lyapunov function possess the capability to relax the stability and positivity analysis of discrete time PFMB control system. The reason is that compared with fixed constant vector $\lambda = [10, 15]^{\mathrm{T}}$ in Remark 5, variable vector $\lambda(\mathbf{x}(k)) = \sum_{1=1}^{3} w_l(\mathbf{x}(k))\lambda_l$ with predefined

$$w_1(x_1(k)) = 1 - \frac{1}{(1 + e^{-(2x_1(k) - 6)})},$$

$$w_2(x_1(k)) = 1 - w_1(x_1(k)) - w_3(x_1(k)), \quad w_3(x_1(k))$$

$$= \frac{1}{(1 + e^{-(2x_1(k) - 14)})},$$

obtained λ_l with $\lambda_1 = [10, 15]^{\mathrm{T}}$, $\lambda_2 = [26.19, 39.29]^{\mathrm{T}}$ and $\lambda_3 = [34.19, 51.29]^{\mathrm{T}}$ based on Theorem 3 through solver is much more capable of finding feasible solutions which leads to relaxed stability analysis.

6 Conclusion

This paper studies the positivity, stability and bounded control analysis of discrete-time PFMB control system based on imperfect premise matching the design concept. Fuzzy co-positive Lyapunov function is proposed to guarantee the stability of discrete-time positive PFMB control system. The formulation of Lyapunov candidate allows the non-negative vectors in terms of system state functions to be contained in Lyapunov function. This improves the generality of Lyapunov function and conservativeness of positivity, stability and bounded control analysis is relaxed compared with common co-positive Lyapunov function. We clearly show via a simulation example that by using the Theorem 3, we can guarantee the positivity, stability and bounded control of discrete-time PFMB control system simultaneously and SOS-based positivity, stability and bounded control conditions can be further relaxed without effecting operating domain for discrete-time PFMB control system.

In the future, the stability and positivity conditions of discrete-time PFMB control system can be further relaxed by considering the information membership functions in the proposed stability and positivity Theorem. Furthermore, the proposed fuzzy co-positive Lyapunov function stability analysis can be employed to control discrete-time positive non-linear system by combining other control methods such as output-feedback and observer-based feedback controller and so on aiming for different kinds of practical application systems.

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